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THE METHOD OF FICTITIOUS ABSORPTION IN PROBLEMS OF THE THEORY OF ELASTICITY FOR AN INHOMOGENEOUS HALF-SPACE†

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The method of fictitious absorption [1–3] is generalized to a class of dynamic mixed problems of the theory of elasticity for a multilayered inhomogeneous half-space. The generalization is based on the use of numerical methods of solving integral equations of the first kind, which enables an exact representation of the symbols of the kernel of the integral operators to be employed and enables one to omit the approximation stage which is necessary when realizing the traditional scheme of the method of fictitious absorption. One thereby completely preserves all the dynamic features of the symbols of the kernel of the integral equation, including the branching points, which leads to a more complete consideration of the dynamic properties of the problem and, consequently, to an increase in the accuracy of the solution obtained in the result.

Previously ([1–3], etc.) the method of fictitious absorption was used to solve integral equations, the symbols of the kernel of which were meromorphic functions, the approximations of which allowed of exact factorization.

1. THE GENERAL SCHEME OF THE METHOD

Consider the integral equation

$$\mathbf{k}q = \int_{-a}^{a} k(x_1 - \xi)q(\xi)d\xi = f(x_1), \quad |x_1| \leqslant a \tag{1.1}$$

$$k(s) = \frac{1}{2\pi}\int_{\Gamma} K(\alpha)e^{i\alpha s}d\alpha \tag{1.2}$$

The function $K(\alpha)$ possesses characteristic properties of the symbols of the kernel of the integral equation which arise when investigating dynamic mixed problems of the theory of elasticity and mathematical physics for a multilayered inhomogeneous half-space: (1) it is even and has, on the real axis, a finite number of branching points, which depend on the type of problem and the properties of the material of the medium, (2) it is meromorphic in the complex plane with cuts, which do not change into one another, situated in the first and third quadrants and connecting the branching points with an infinitely distant point, (3) it has, one the real axis, a finite number of zeros $\gamma_k (k = 1, 2, \ldots, n_2)$ and poles $z_k (k = 1, 2, \ldots, n_1)$, which depend on the frequency, and also a denumerable set of complex zeros and poles with condensation points in certain sectors containing the imaginary axis, and (4) at infinity it can be represented in the form

$$K(\alpha) = c|\alpha|^{-1}[1 + O(\alpha^{-1})]$$

Integral equation (1.1) is uniquely solvable for any doubly continuous differentiable function $f(x_1)$ [4]; the location of the contour Γ in integral (1.2) ensures that the radiation conditions are satisfied.

We will introduce the functions

$$\Pi(\alpha) = \prod_{k=1}^{M} (\alpha^2 - \gamma_k^2)(\alpha^2 - z_k^2)^{-1} \text{ and } K_0(\alpha) = K(\alpha)\Pi^{-1}(\alpha)$$

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where z_k $(k = 1, 2, \ldots, n_1)$ and γ_k $(k = 1, 2, \ldots, n_2)$ are real poles and zeros of the function $K(\alpha)$, while the remaining z_k $(k = n_1 + 1, \ldots, M)$ and γ_k $(k = n_2 + 1, \ldots, M, M \geqslant \max\{n_1, n_2\})$ are complex poles and zeros of $K(\alpha)$ lying in the strip $|\mathrm{Im}\alpha| \leqslant E_0$.

It can be seen that the function $K_0(\alpha)$, which in general depends on the frequency, includes all singularities of the function $K(\alpha)$ ignored in $\Omega(\alpha)$ and primarily the branching points on the real axis. The asymptotic properties of the functions $K(\alpha)$ and $K_0(\alpha)$ are identical since $\Pi(\infty) = 1$ by construction.

Using the method of fictitious absorption we will represent the symbol of the kernel of integral equation (1.1) in the form

$$K(\alpha) = K_0(\alpha)\Pi(\alpha) \tag{1.3}$$

Remark 1. Previously [1–3], when using the method of fictitious absorption, the approximate representation (1.3) of the function $K(\alpha)$ was employed, which allows of exact factorization, in which $K_0(\alpha) = c(\alpha^2 + B^2)^{-1/2}$, where $B \gg 1$ is a specified parameter.

Definition. We will call the least set, closed in Ω, outside of which $q(x_1) = 0$, the carrier of the function $q(x_1)$, specified in the region Ω.

Lemma [1]. Suppose the function $q(x_1) \in \mathrm{L}_p[-a, a], p > 1$ has a carrier in the range $[-a, a]$. In order that the function ($V(\alpha)$ and $V^{-1}(x_1)$ are the direct and inverse Fourier transformation operators) should possess the same property, it is necessary and sufficient for the identify $\mathbf{V}(\alpha)q \equiv 0$ to hold in the polar set of the function $\Pi(\alpha)$ (i.e. when $\alpha = \pm z_k, k = 1, 2, \ldots, M$).

We will represent the solution of Eq. (1.1) in the form [1]

$$q(x_1) = q_0(x_1) + \varphi(x_1) \tag{1.4}$$

We will require that the relations

$$V(\alpha)q = V(\alpha)\varphi, \quad V(\alpha)q_0 = 0, \quad \alpha = \pm z_k, \quad k = 1, 2, \ldots, M \tag{1.5}$$

must be satisfied.

Remark 2. Here and henceforth the poles z_k and zeros γ_k are assumed to be simple. There are no fundamental difficulties in extending our analysis to the case of multiple zeros and poles. It is only necessary to take into account that, in this case, representation (1.3) and the form of relations (1.5) are changed [1].

Substituting expression (1.4) into integral equation (1.1), we convert it to the form

$$\mathbf{k}q_0 = f - \mathbf{k}\varphi \tag{1.6}$$

We further introduce the function [1]

$$t(x_1) = \mathbf{V}^{-1}(x_1)T(\alpha), \quad T(\alpha) = \Pi(\alpha)Q_0(\alpha), \quad Q_0(\alpha) = \mathbf{V}(\alpha)q_0(x_1) \tag{1.7}$$

and use it as the new unknown.

The problem of solving (1.6), taking relations (1.3) and (1.7) into account, thereby reduces to solving the integral equation

$$\int_{-a}^{a} k_0(x_1 - \xi)t(\xi)d\xi = -\mathbf{k}\varphi + f \tag{1.8}$$

Remark 3. It follows from the construction that integral equation (1.8) is equivalent to (1.6), i.e. in this case, the need to use the perturbation theorem [1] is eliminated.

We will assume that the solution of integral equation (1.8) has been constructed and the function $t(x_1)$ has been obtained. Relations (1.7) enable the function $q_0(x_1)$ to be re-established in the form

$$q_0(x_1) = \mathbf{V}^{-1}(x_1)\Pi^{-1}(\alpha)\mathbf{V}(\alpha)t(x_1) \tag{1.9}$$

It follows from the lemma that in order for the function $q_0(x_1)$ to belong to $L_p[-a, a], p > 1$, and have a carrier in $[-a, a]$, the following relations must be satisfied

$$\mathbf{V}(\alpha)t(x_1) = 0, \quad \alpha = \pm\gamma_k, \quad k = 1, 2, ..., M \tag{1.10}$$

i.e. the function $t(x_1)$ must contain an arbitrariness, the degree of which is determined by the number of conditions (1.10). This arbitrariness must be put into the function $\varphi(x_1)$, which forms part of solution (1.4).

Using relations (1.4) and (1.9), we construct the Fourier transformant of the solution of the initial integral equation (1.1) in the form

$$Q(\alpha) = T(\alpha)\Pi^{-1}(\alpha) + \mathbf{V}(\alpha)\varphi \tag{1.11}$$

To obtain the integral characteristic of the problem (for example, the reaction of the base to the action of a punch), it is sufficient to put $\alpha = 0$ in (1.11). To construct $q(x_1)$ – the solutions of the initial integral equation (1.1), it is necessary to apply an inverse Fourier transformation to relation (1.11).

2. REALIZATION OF THE METHOD

Earlier we introduced a function $\varphi(x_1)$ containing an arbitrariness (its form was not specified), which must be constructed from the values of the functionals (1.10). In the final expressions $\varphi(x_1)$ is present under the integral operator sign, by choosing which, one/can use a fairly wide class of functions [1]. Here we will use as $\varphi(x_1)$ a system of Dirac δ-functions

$$\varphi(x_1) = \sum_{k=1}^{2M} C_k\delta(x_1 - x_1^k) \tag{2.1}$$

where x_1^k are the coordinates of points which divide the segment $[-a, a]$ into equal parts. It can be verified that the function $\varphi(x_1)$ satisfied relations (1.5).

After substituting expression (1.7) into integral equation (1.1) and taking Eq. (2.1) into account, the integral equation takes the form

$$\begin{gathered}\int_{-a}^{a} k(x_1 - \xi)q_0(\xi)d\xi = \sum_{k=1}^{2M} C_k f_k(x_1) + f_0(x_1) \\ f_k(x_1) = -k(x_1 - x_1^k), \quad f_0(x_1) = f(x_1)\end{gathered} \tag{2.2}$$

Bearing expressions (1.7) and (1.3) in mind, we can convert integral equation (2.2) to an integral equation in the new unknown $t(x_1)$

$$\int_{-a}^{a} k_0(x_1 - \xi)t(\xi)d\xi = \sum_{k=1}^{2M} C_k f_k(x_1) + f_0(x_1) \tag{2.3}$$

According to the lemma, the function $t(x_1)$ must satisfy functionals (1.10), which we will represent in a form that is more convenient for subsequent analysis

$$T(\alpha) = 0, \quad \alpha = \pm\gamma_k, \quad k = 1, 2, ..., M \tag{2.4}$$

Equalities (2.4) represent a system of $2M$ equations in $2M$ unknowns C_k, which occur in (2.1), and which, taking (1.4) into account, closes the problem and enables the Fourier transformant of the solution of integral equation (1.1) to be represented in the form

$$Q(\alpha) = T(\alpha)\Pi^{-1}(\alpha) + \sum_{k=1}^{2M} C_k e^{i\alpha x_1^k} \tag{2.5}$$

3. NUMERICAL REALIZATION OF THE METHOD

Using the superposition principle, we can represent the solution of Eq. (2.3) in the form

$$t(x_1) = t_0(x_1) + \sum_{k=1}^{2M} C_k t_k(x_1) \tag{3.1}$$

where the functions $t_k(x_1)$ ($k = 0, 1, \dots, 2M$) satisfy the equations

$$\mathbf{k}_0 t_k = \int_{-a}^{a} k_0(x_1 - \xi) t_k(\xi) d\xi = f_k(x_1), \quad |x_1| \leq a \tag{3.2}$$

We will introduce two systems of coordinate functions $\psi_p(x_1)$, $\varphi_p(x_1)$ ($p = 1, 2, \dots, N$) and put

$$t_k(x_1) = \sum_{p=1}^{N} \beta_k^p \psi_p(x_1), \quad k = 0, 1, \dots, 2M \tag{3.3}$$

After substituting expressions (3.3) into Eqs (3.2) we obtain the system of equations

$$\sum_{p=1}^{N} \beta_k^p \int_{-a}^{a} k_0(x_1 - \xi) \psi_p(\xi) d\xi = f_k(x_1), \quad |x_1| \leq a \tag{3.4}$$

Applying the weighting procedure of system (3.4) to the coordinate functions $\varphi_1(x_1)$ ($l = 1, 2, \dots, N$), we arrive at the need to solve $2M + 1$ systems of algebraic equations

$$\mathbf{A}\mathbf{B}_k = \boldsymbol{F}_k, \quad k = 0, 1, \dots, 2M \tag{3.5}$$

where

$$\mathbf{A} = \left\| A_{pl} \right\|_{p,l=1}^{N}, \quad A_{pl} = \int_{\Gamma} K_0(\alpha) \Psi_p(\alpha) \Phi_l^*(\alpha) d\alpha$$

$$\mathbf{F}_k = \{f_k^l\}_{l=1}^{N}, \quad f_k^l = \int_{\Gamma} K(\alpha) \Phi_l(\alpha) e^{-i\alpha x_1^k} d\alpha$$

$$\mathbf{B}_k = \{\beta_k^p\}_{p=1}^{N}, \quad f_0^l = \int_{-a}^{a} f(x_1) \varphi_l(x_1) dx_1$$

$\Psi_p(\alpha)$ and $\Phi_l(\alpha)$ are the Fourier transformations of the functions $\psi_p(x_1)$ and $\varphi_l(x_1)$, and $(\ )^*$ is a complex-conjugate quantity.

Suppose the matrix $\mathbf{A}$ is constructed and from it $\mathbf{A}^{-1}$. Then the solutions of systems (3.5) can be represented in the form

$$\mathbf{B}_k = \mathbf{A}^{-1} \mathbf{F}_k \tag{3.6}$$

It follows from relation (3.6) that the main burden of solving the $2M + 1$ systems of algebraic equations (3.5) lies in constructing the matrix $\mathbf{A}$ and calculating $\mathbf{A}^{-1}$. By construction it is sufficient to do this once. Further, by successive use of relation (3.6) one can calculate all the vectors $\mathbf{B}_k$, the components of which are the coefficients β_k^p ($p = 1, \dots, N$; $k = 0, 1, \dots, 2M$).

To construct the solution of integral equation (2.3) we will represent solution (3.1) in the form

$$T(\alpha) = T_0(\alpha) - \sum_{k=1}^{2M} C_k T_k(\alpha), \quad T_k(\alpha) = \sum_{p=1}^{N} \beta_k^p \Psi_p(\alpha) \tag{3.7}$$

The constants C_k, which occur in (2.5), can be found from the system

$$T_0(\pm\gamma_n) - \sum_{k=1}^{2M} C_k T_k(\pm\gamma_n) = 0, \quad n = 0, 1, \dots, 2M$$

which is obtained when expressions (3.7) are substituted into relations (2.4).

Remark 4. If it is necessary to calculate the function $q(x_1)$ – the solution of the initial integral equation (1.1), one cannot use representation (2.5), since, after applying an inverse Fourier transformation to (2.5), the final expressions will contain a generalized function. In this case, either the function $\varphi(x_1)$ must be chosen from the class L_p, $p > 1$, or the function $t(x_1)$ must be introduced in such a way [1], that the function $\varphi(x_1)$ is present under the integral operator sign in the final expressions for $q(x_1)$.

5. The method can be extended, without particular difficulty, to the class of spatial axisymmetric problems of the theory of elasticity. In this case, it is sufficient in the constructions to change to the new variable $u = \sqrt{\alpha_1^2 + \alpha_2^2}$.

4. THE DYNAMICS OF A PUNCH ON THE SURFACE OF AN INHOMOGENEOUS HALF-SPACE

We will consider, as an example, the problem of the oscillations of a rigid punch on the surface of a structurally inhomogeneous medium, which takes the form of a prestressed layer $0 \leqslant x_3 \leqslant h$ on the surface of a prestressed half-space $x_3 \leqslant 0$. We will assume that the half-space is more rigid, i.e. $\lambda_s < \lambda_p$, $\mu_s < \mu_p$ (the subscript s denotes the parameters of the layer while the subscript p denotes the parameters of the half-space), and the medium is isotropic in its natural state. The punch occupies the region $|x_1| \leqslant a$ in plan and executes steady vertical translational oscillations. There is no friction in the contact area.

We will assume that the stress–strain state is homogeneous and we will consider the following:

1) the "surface" stress–strain state – a prestressed layer on a stress-free half-space (the layer is first stretched or compressed, and is then combined with the half-space);

2) a "deepened" stress–strain state – a stress-free layer on a prestressed half-space (the half-space is first stretched or compressed, and is then connected to the layer).

The boundary-value problem is described by the linearized equations of motion [5]

$$\nabla \cdot \mathbf{\Theta}^{(n)} = \rho^{(n)} \ddot{\mathbf{u}}^{(n)}, \quad n = 1, 2, \tag{4.1}$$

with boundary conditions on the surface

$$x_3 = h, \quad \mathbf{n} \cdot \mathbf{\Theta}^{(1)} = \begin{cases} \mathbf{q}(x_1)\exp(-i\omega t), & |x_1| \leqslant a \\ 0, & |x_1| > a \end{cases} \tag{4.2}$$

where $\Theta^{(n)}$ are linearized tensors, which play the role of the Cauchy tensor in the linear theory of elasticity, $\rho^{(n)}$ is the density of the material, $\mathbf{u}^{(n)}(x_1, x_3, t) = \{u_1^{(n)}, u_2^{(n)}, u_3^{(n)}\}$ is the vector of displacements of points of the layer ($n = 1$) or the half-space ($n = 2$) respectively, $\mathbf{n}$ is the vector of the normal to the surface of the medium and $\mathbf{q}(x_1)$ is an unknown vector of the contact stresses, the components of which, when there is no friction in the contact area and taking into account the translational nature of the oscillations, have the form $\{0, 0, q_{30}\}$.

The problem is closed by the conditions that the layer is joined to the half-space

$$x_3 = 0, \quad \mathbf{u}^{(1)} = \mathbf{u}^{(2)}, \quad \mathbf{t}_n^{(1)} = \mathbf{t}_n^{(2)} \tag{4.3}$$

and the condition at infinity

$$\mathbf{u}^{(2)} \downarrow 0, \quad x_3 \to -\infty \tag{4.4}$$

Here $\mathbf{t}_n^{(k)}$ are the stress vectors at the interface between the layer and the half-space. The form of the tensors $\Theta^{(n)}$, which occur in expressions (4.1) and (4.2), is extremely lengthy and will not be given here. Their components were given earlier for an elastic Murnagan potential in [6–8]. In view of the steady nature of the oscillations the time factor will henceforth be omitted.

Boundary-value problem (4.1)–(4.4) reduces to the solution of integral equation (1.1), in which $f(x_1) = u_{30}^{(1)}$ (the displacement of the points of the surface of the medium – the amplitude of the punch oscillations), and $q(x_1) = q_{30}$ (the unknown contact stresses).

When the stress–strain state of the layer ($n = 1$) or the half-space ($n = 2$) is specified by the conditions

$$\sigma_{11}^{(n)0} \neq \sigma_{22}^{(n)0} \neq \sigma_{33}^{(n)0}$$

we have

$$K(\alpha,\omega)=\sum_{k=1}^{2}\left[\Delta_{kp}\operatorname{ch}\sigma_k^{(1)}h+\Delta_{k+2,p}\operatorname{sh}\sigma_k^{(1)}h\right] \tag{4.5}$$

$$\begin{aligned}
&\Delta_{mj}=\Delta_{mj}^0/\Delta^0,\quad m,j=1,2,\ldots,6,\quad \Delta^0=\det\left\|T_{mj}\right\|_{m,j=1}^{6}\\
&T_{1k}=l_{1k}^{(1)}\operatorname{ch}\sigma_k^{(1)}h,\quad T_{1,k+2}=l_{1k}^{(1)}\operatorname{sh}\sigma_k^{(1)}h,\quad T_{1,k+4}=0\\
&T_{2k}=l_{3k}^{(1)}\operatorname{sh}\sigma_k^{(1)}h,\quad T_{2,k+2}=l_{3k}^{(1)}\operatorname{ch}\sigma_k^{(1)}h,\quad T_{2,k+4}=0\\
&T_{3k}=0,\quad T_{3,k+2}=f_k^{(1)},\quad T_{3,k+4}=-f_k^{(2)}\\
&T_{4k}=1,\quad T_{4,k+2}=0,\quad T_{4,k+4}=-1\\
&T_{5k}=l_{1k}^{(1)},\quad T_{5,k+2}=0,\quad T_{5,k+4}=-l_{1k}^{(2)}\\
&T_{6k}=0,\quad T_{6,k+2}=l_{3k}^{(1)},\quad T_{6,k+4}=-l_{3k}^{(2)};\quad k=1,2
\end{aligned} \tag{4.6}$$

$$\begin{aligned}
&l_{1k}^{(n)}=\chi_{3113}^{(n)}\sigma_k^{(n)}f_k^{(n)}-i\alpha\chi_{1313}^{(n)},\quad S_{1k}^{(n)}=B_{32}^{(n)}H_{1k}^{(n)}+\alpha^2B_{13}^{(n)}B_{23}^{(n)}\\
&l_{3k}^{(n)}=\chi_{3333}^{(n)}\sigma_k^{(n)}-i\alpha\chi_{3311}^{(n)}f_k^{(n)},\quad S_{3k}^{(n)}=B_{12}^{(n)}H_{3k}^{(n)}-\sigma_k^{(n)^2}B_{13}^{(n)}B_{23}^{(n)}\\
&H_{jk}^{(n)}=\chi_{3jj3}^{(n)}\sigma_k^{(n)^2}-(\chi_{kjjk}^{(n)}\alpha^2-\rho^{(n)}\omega^2),\quad j=1,2,3\\
&B_{jk}^{(n)}=\chi_{jjkk}^{(n)}+\chi_{jkjk}^{(n)},\quad f_k^{(n)}=-i\alpha S_{3k}^{(n)}(\sigma_k^{(n)}S_{1k}^{(n)})^{-1}
\end{aligned} \tag{4.7}$$

$$\begin{aligned}
&\chi_{lmsp}^{(n)}=\delta_{ls}\delta_{mp}s_{lm}^{1n}+\delta_{ms}\delta_{lp}\nu_l^2s_{lm}^{2n}+\delta_{lm}\delta_{sp}s_{ls}^{3n}\\
&s_{lm}^{1n}=2J_n^{-1}[-\psi_{0n}+\psi_{2n}\upsilon_{nl}^2\upsilon_{nm}^2],\quad s_{lm}^{2n}=2J_n^{-1}[\psi_{1n}+\psi_{2n}(\upsilon_{nl}^2+\upsilon_{nm}^2)]\\
&s_{lm}^{3n}=4J_n^{-1}\sum_{M=0}^{2}\sum_{N=0}^{2}V_{MN}^{(n)}\upsilon_{nl}^{2M}\upsilon_{nm}^{2N}
\end{aligned} \tag{4.8}$$

In formulae (4.6)–(4.8) the subscripts n and k take the values 1 and 2, $\Delta_{lp}^0(l,p=1,2,\ldots,6)$ are the cofactors of the elements T_{lp}, $\rho^{(n)}$ is the density, υ_{nm} is the relative extension of the fibres along the x_m axis, J_n is the metric factor of the layer ($n=1$) or the half-space ($n=2$), δ_{mp} is the Kronecker delta, and the coefficients ψ_{nL} and $V_{LM}^{(n)}(L,M=0,1,2)$ depend on the form of the elastic potential. Their form, for certain special cases, was given previously in [5–8]. The quantities $\sigma_k^{(n)}$ are found from the characteristic equation

$$H_{1k}^{(n)}H_{3k}^{(n)}+\alpha^2\sigma_k^{(n)^2}B_{13}^{(n)^2}=0$$

A numerical analysis was carried out for a layer (bronze [5]), rigidly bound to a half-space (35 KhGSA steel [5]). The thickness of the layer $h=1$ and the initial stressed state was defined by the conditions $\upsilon_1=\upsilon_2=\upsilon_3\ 1\pm 0.005$.

In Fig. 1 we show curves of the real zeros (the dashed curves) and poles (the continuous curves) of the function $K(\alpha)$ (4.5) as a function of the dimensionless frequency ($\varkappa_2=\sqrt{\rho^{(2)}/\mu^{(2)}}\,\omega h$) when there are no initial stresses. The form of the curves, and also the number of real zeros and poles, which increase with frequency, are characteristic for problems in the case of a multilayered half-space. Strict alternation of the zeros and poles occurs in this case, which is ensured by the uniqueness of the solution of the integral equations [4].

As calculations have shown, a change over a fairly wide range in the value of the initial stresses does not change the qualitative form of the distribution of the zeros and poles, but leads to a considerable change in their value. It has been established that the spectral properties of the problem depend very much on which region of the composite medium is subject to the action of initial stresses. Localization of the stress–strain state in the half-space produces a considerable deformation of the dispersion curves and a frequency shift of these curves. Stretching of the half-space (the layer is unstressed) leads to an

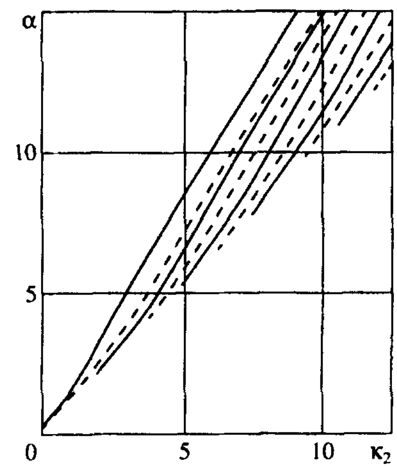

Fig. 1

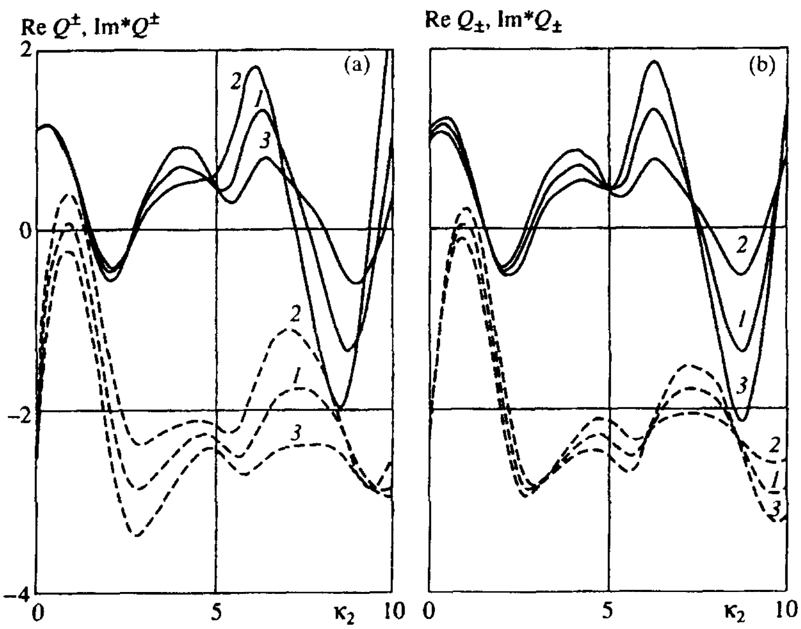

Fig. 2

increase in the values of the zeros and poles, and the values of the critical frequencies (the frequencies at which higher modes occur) increase. If we take into account the fact that the poles z_k are connected with the phase space of the surface wave V_k by the relation $V_k = \omega z_k^{-1}$, stretching of the half-space leads to a reduction in the phase velocities of all the modes, proportional to the initial stresses. Compression of the half-space leads to the opposite result.

Localization of the stress–strain state in the layer (the half-spaces is unstressed) affects the dispersion characteristics of the problem somewhat differently. Stretching of the layer leads to a reduction in the values of the first branch of the poles, but the values of subsequent branches of the poles, as well as the values of the points of their emergence, increase. Stretching of the layer thereby leads to an increase in the phase velocity of the first mode and a reduction in the velocities of higher modes of surface waves. Changes in the velocities are proportional to the initial stresses. Compression of the layer leads to the opposite result.

The effect of localization of prestresses on the dynamic stiffness of the composite medium is illustrated in Fig. 2, in which we show graphs of the functions $\mathrm{Re}Q^{\pm}$ (the continuous curves), $\mathrm{Im}^*Q^{\pm} = \varkappa_2^{-1}\,\mathrm{Im}\,Q^{\pm} + 6$ (the dashed curves) in Fig. 2(a) and $\mathrm{Re}Q_{\pm}$ (the continuous curves), $\mathrm{Im}^*Q_{\pm} = \varkappa_2^{-1}\,\mathrm{Im}\,Q_{\pm} + 6$ (the dashed curves) in Fig. 2(b). The functions $Q^{\pm}$ and $Q_{\pm}$ correspond to the dynamic stiffness of the medium

$$Q_0 = \int_{-a}^{a} q_{30}(x_1)dx_1$$

($q_{30}(x_1)$ is the solution of Eq. (1.1) when $f(x_1) = 1$) for preliminary compression (the minus sign) or preliminary tension (the plus sign) of the layer (the superscript) or the half-space (the subscript). The calculations were carried out for $a = 1$.

Curves 1 correspond to the natural state, curves 2 correspond to tension and curves 2 correspond to compression of the corresponding region of the composite medium. It follows from the groups that $\mathrm{Re}Q_0$ and Im^*Q_0, when there are no initial stresses, have an oscillatory form, due to the non-uniformity of the medium, on which localization of the stress–strain state has a considerable influence. Compression of the layer (curves 3 in Fig. 2a) as well as tension of the half-space (curves 2 in Fig. 2b) reduce the non-uniformity of the multilayered structure. Conversely, tension of the layer, as well as compression of the half-space, increases the non-uniformity of the medium. This confirms the peculiarity, pointed out earlier in [6–8], of the effect of the stress–strain on the dynamic stiffness of the medium: compression of the layer or of the homogeneous half-space leads to an increase in their dynamic stiffness, while tension leads to its reduction.

It also follows from the graphs that when the stress–strain state is localized in the half-space, frequencies of maximum and minimum effect of the initial stresses on the reaction of the medium exist. There are no such frequencies when the stress–strain state is localized in the layer. Hence, in a structurally inhomogeneous medium the features detected previously in [6–8] when the dynamic properties of a prestressed layer [8] and a homogeneous half-space [6, 7] were investigated are preserved: the presence in the problem for a half-space of frequencies of maximum and minimum influence of the stress–strain state on the reaction of the medium and the absence of such frequencies in problems for a layer.

It should be noted that compression of the layer leads to an-increase in the level of radiation of energy from the contact area, while tension in the layer leads to its reduction at all frequencies. Localization of the initial stresses in the half-space also affects the level of radiation from the contact area, but this change depends on the frequency.

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